

Logic for Justice

**An Introduction to Propositional and First-Order Logic,
Emphasizing their Relevance to Political Reform**

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For Sophie

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Preface (For Teachers)

This textbook is a formally rigorous introduction to the basics of propositional logic and first-order logic. It motivates the study of those logical systems by drawing connections between (i) logic, (ii) natural language arguments, and (iii) the sorts of social and political issues that students generally find important. As such, this textbook has three main innovative features.

First, the pedagogical presentation starts with natural language, and then introduces logical languages from there. The language of propositional logic, for instance, is introduced one symbol at a time. Each new symbol is explained in terms of, among other things, various natural language expressions with approximately similar meanings. Only then is the next new symbol introduced. After working through all the symbols in this way, a summary of the resulting formal language is provided.

This is, in my experience, a much better method for teaching logic than the method used in most logic textbooks. Those other textbooks often introduce the entire formal language at once, right away; connections between that formal language and natural languages, if discussed at all, are only made much later. And for straightforward pedagogical reasons, that is somewhat odd. It is like teaching Mandarin to English-speakers by first introducing all 100,000+ characters of Mandarin along with the standard rules of Mandarin grammar, and only later connecting everything to English vocabulary and grammar in English. That would be an odd way to teach Mandarin to native English speakers. And it is similarly odd to teach logic by first defining the relevant formal languages in full precision and generality, and only later connecting those formal languages to the natural languages which students already know. Better to start in familiar territory and move into the unfamiliar from there, than to start in unfamiliar territory and frantically search for something familiar.

Second, this textbook motivates logic by, among other things, connecting it to social justice. The connection comes by way of natural language arguments. If students care about social justice and political reform, then they should care about giving good arguments. Propositional logic and first-order logic provide two reasonable theories of one way for natural language arguments to be good. So learning logic will help students pursue their social and political goals.

Note how qualified the above claims, about the connection between logic

and natural language arguments, actually are. I suggest only that each system of logic, presented in this textbook, provides *one* reasonably attractive theory of *one* way for an argument to be good: basically, being valid is one way of being good, and propositional logic and first-order logic provide reasonably attractive theories of what it takes to be valid. There are, of course, other reasonably attractive theories of validity; I discuss this occasionally in what follows. And there are, of course, other ways for arguments to be good: I discuss this occasionally in what follows too. So to disagree with my qualified claims, you would have to argue that one of the systems of logic, presented in this textbook, fails to provide *any* reasonably attractive theory of *any* way for an argument to be good. Though that view is, of course, worth exploring, the pedagogical gains of setting it aside—and motivating logic by drawing connections to good argumentation and social justice—far outweigh the costs.¹

Third, compared to other logic textbooks, this one spends more time emphasizing the virtues of propositional logic and first-order logic. All too often, logic textbooks—and logic teachers too—emphasize the vices of those logical theories, while spending little to no time explaining what makes those theories attractive. This is, of course, poor pedagogy: it confuses students, making them wonder why propositional logic and first-order logic are worth studying. This is also self-undermining: it is hard for students to appreciate the problems that a particular theory of logic faces, before first appreciating why that theory was developed in the first place. So in this textbook, I adhere to the following basic pattern: start by discussing the virtues of thus-and-so features of propositional logic and first-order logic, and only later discuss those features' vices.

¹Besides, coherent arguments for that view are extremely difficult to formulate. It is hard to formulate non-circular, logically respectable arguments for the view that systems like propositional logic and first-order logic do not provide even reasonably decent approximations of the logical structures of some sizeable fragment of natural language arguments. Such arguments risk presupposing the very sorts of logical structures which propositional logic and first-order logic model; so for reasons which have been known since the time of Aristotle (ca. 350 B.C.E./1998), such arguments are often self-undermining.

*When logic and proportion
have fallen sloppy dead,
and the White Knight is talking backwards
and the Red Queen's "Off with her head!"
remember what the dormouse said:
"Feed your head! Feed your head!"
— Grace Slick*